## MD'2010, zadanie domowe nr 1

**termin: 2010-03-04**

Oblicz

$$\sum_{k=0}^{\infty} \frac{1}{(k+1)(k+3)(k+5)} \ .$$

# Zadanie 1

Xilexio

Zauważmy, że:

$$\frac{1}{(k+1)(k+2)(k+3)} = \frac{1}{4}\left(\frac{1}{(k+1)(k+3)} - \frac{1}{(k+3)(k+5)}\right) =$$
$$= \frac{1}{8}\left(\left(\frac{1}{k+1} - \frac{1}{k+3}\right) - \left(\frac{1}{k+3} - \frac{1}{k+5}\right)\right) = \frac{1}{8}\left(\frac{1}{k+1} + \frac{1}{k+5} - \frac{2}{k+3}\right)$$

Dla n $\geq 5$ obserwujemy ładne skracanie się w sumach częściowych:

$$8\sum_{k=0}^{n}\frac{1}{(k+1)(k+2)(k+3)} = 8S_n = \frac{1}{1} + \frac{1}{5} - \frac{2}{3} +$$
$$+\frac{1}{2} + \frac{1}{6} - \frac{2}{4} +$$
$$+\frac{1}{3} + \frac{1}{7} - \frac{2}{5} +$$
$$+\frac{1}{4} + \frac{1}{8} - \frac{2}{6} +$$
$$+ ... +$$
$$+\frac{1}{n-2} + \frac{1}{n+2} - \frac{2}{n} +$$
$$+\frac{1}{n-1} + \frac{1}{n+3} - \frac{2}{n+1} +$$
$$+\frac{1}{n} + \frac{1}{n+4} - \frac{2}{n+2} +$$
$$+\frac{1}{n+1} + \frac{1}{n+5} - \frac{2}{n+3} =$$
$$= \frac{1}{1} - \frac{2}{3} + \frac{1}{2} - \frac{2}{4} + \frac{1}{3} + \frac{1}{4} +$$
$$+\frac{1}{n+2} + \frac{1}{n+3} + \frac{1}{n+4} - \frac{2}{n+2} + \frac{1}{n+5} - \frac{2}{n+3} =$$
$$= \frac{11}{12} + \frac{1}{n+2} + \frac{1}{n+3} + \frac{1}{n+4} - \frac{2}{n+2} + \frac{1}{n+5} + \frac{2}{n+3} \xrightarrow{n\to\infty} \frac{11}{12}$$

$$\sum_{k=0}^{\infty}\frac{1}{(k+1)(k+2)(k+3)} = \lim_{n\to\infty} S_n = \frac{11}{96}$$